Δ calculus — Mandate[1]

Last edited: Tuesday, May 3, 2016

I • Background

A. Foundational calculi have played a crucial role in the development of science. The differential calculus was critical to the development of physics; formal logic (the predicate calculus) to the study of reasoning & entailment; algebra to the arithmeticisation of geometry; etc. Getting a calculus "right" is hugely beneficial in allowing us to register appropriately, and thereby come to understand, diverse subject matters.[2]
   1. Cf. the remarks on a "kernel calculus" in the introduction to IA·1.

B. What it is to be "good" (better or worse, for the purposes at hand) is epistemologically & ontologically subtle.[3]

C. Ontic commitments
   1. In terms of ontological/metaphysical commitments—in how they register (or support registration) of the world—calculi effectively make or impose a 3-way distinction (is this right?) among :[4]
      a. What is embodied or "built in" to the calculus itself—and can therefore be assumed to be true (or at least claimed) of all subject domains described in it (unless modified by meta-level commentary, possible in a reflective calculus, or so at least I claim);
      b. What is then "said" in the calculus—in specific descriptions, theories, or claims; and
      c. What is effectively "disappeared" or removed from the discussion, in virtue of falling outside (a).
   2. There is thus effectively a 2-way "division of labour" in what can be said—between (a) & (b)—whereas category (c) includes what is not and cannot be said.[5]

D. A kernel calculus
   1. What is (ontologically) "built in" will typically be embodied in two ways:
      a. Categorematic: primitives (terms, roughly) that cannot be defined within the calculus without being used in the definition (even at a meta-level). There is a question (I don't know the answer offhand; tai!) as to the status of categorematic semantical terms vs. syntactical (but of course semantic) ones.
      b. Syncategorematic structures & operations (operators, forms of composition, etc.).
   2. We call such characteristics **kernel** (rather than primitive), and thus speak of the **kernel (ontological) commitments** embodied in the calculus' structure
      a. Or: *kernel content*, *implicit content*, *automatic* content?
      b. Thus the differential calculus (≡∂c) makes a kernel commitment to the fact that the regularities it can be used to express will be formulated as derivatives & integrals of—usually temporally—dependent measure variables. Even purely mathematical calculi without obvious concrete subject matters, such as algebra & set theory, still typically embody specific ad/or particular kernel commitments. (Say what they are!)
   3. What is "said" in the calculus—i.e., the "meaning" or "content" of its descriptions—we will call **constructed commitments**.
      a. Or: *constructed content*, or explicit content?
   4. The net registration of the world embodied in a set of Δ calculus structures will consist of its kernel & constructed commitments.

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[1] Based on a file called "Δ calculus — Mandate · 01" from 2008 · October · 13;

[2] Cf. Newton's early work on a calculus based on the *radius of curvature* of a function, at a given value, as a basis in terms of which to frame the laws of motion—a project that didn't work out very well. The shift to the less-geometrically evident notion sof *slope* was radically more congenial to the framing of the world's physical regularities.

[3] It of course on the purposes for which the calculus is needed or used.

[4] Calculi are thus a kind of language, though I make no claim here as to *what* kind. Among other things, calculi are clearly more "formal" than natural languages, implying that the divide between "the language itself" (its kernel commitments) and "what is said in the language" (its constructive commitments) is sharper than in the natural case. Note, however, that as with everything, what constitutes "Δ CALCULUS itself" versus an instantiation of Δ CALCULUS, extended with various constructed structures & commitments, is not an intrinsic matter; what is the case will depend on how the various systems are respectively registered (including the denotation of the name 'Δ CALCULUS', which again is not fixed by the system's design).

[5] Cf. the "impossible zone" in Haugeland's "Truth & Rule-Following."

5. What is *not* sayable, I suppose we could call **prohibited content**.

E. Design
   1. Common use of a calculus helps facilitate the comparison & contrasting of divergent claims or registrations expressed within it. Sciences (such as contemporary syntactical linguistics) in which there is not a common calculus, with each theory then being expressed in its own formalism, make such comparisons vexatious.
   2. Judicious allocation of ontic commitments across the kernel/constructed divide is one of the most normative criteria on a calculus' worth.
      a. Excessively general calculi (with little kernel structure & commitment) provide the theorist with no help in registering the world.
      b. Conversely, calculi can constrain imagination to regularities expressible in their terms; theories or suggestions that violate their kernel commitments can be difficult to communicate or express, often leading to misunderstanding.[6]

F. The following 5 calculi are (±) among the most important to have been developed to date:
   1. **Algebra**
   2. **Differential calculus** (built on top of algebra)
   3. **Set theory**
   4. **λ-calculus**
   5. **Formal logic** (propositional, predicate & quantificational)

G. Some mathematical formalism & systems (such as dynamical systems theory [DST]) receive a lot of development for use as a framework in terms of which to register phenomena, but aren't themselves calculi (DST uses algebra & the differential calculus)

II • Principles

A. The Δ calculus is a calculus of *description* (as well as operation), designed to satisfy a dozen fundamental principles:

	Property		**Description**
P1)	*Perspectival identity*	·	Identity is not taken to be an intrinsic property of *anything* (including the Δ calculus structures themselves). Rather, descriptions that depend on issues of identity—of property & type as well as object or individual[7]—must "apply" individuation criteria as part of their meaning or content. The issue of whether that which is registered "satisfies" the relevant identity criteria is part of what determines how & whether the description "fits" the world—meaning that Δ calculus descriptions & terms, like sentences in traditional calculi, have "success conditions."
P2)	*Deferential semantics*	·	In a very broad sense, Δ calculus structures are reminiscent of representations, in containing or conveying information *about something* (typically, something *else;* only in extremely rare cases will they themselves be that of which they speak). In Δ calculus, however, we say that descriptions **register** their subject matters.[8] Although registrations, including how they are used, shoulder responsibility for (i.e., are the locus of the determination of) how they register their subject matters, and although normative considerations that stem from this use, it is nevertheless presumed that it is the world (that which they register) that is ultimately the truth maker. In this sense of being normatively **deferential** to the world the semantics has a classical flavour.

[6] In this sense calculi (and perhaps all languages) establish a particularly simple typology of the "domain of comprehensibility" within which the possible & the actual can be distinguished from the impossible but conceivable, as opposed in term to the inconceivable.

[7] Feature-placing requires feature identity, though not object identity; but the feature identity will itself be perspectival (so whether, if it is raining in place α, that means that it is or is not pouring at place β, will depend on whether the notion of 'raining' that it is at place α is a "superfeature" of "pouring" or a sibling feature.

[8] No Δ calculus structure, therefore, will *be* the name of a book, or the length of a list, or the address of a cell (though there may be structures that register that name, length, & address in canonical (normal-form) ways.

			There is a sense in which nothing "is" as it is registered to be, and also a sense (if the registration is "worthy" that things *are* as they are registered to be. The point is that nothing is *exhaustively* its registration; there is always more to anything α than it is or can be registered.
P3)	*Contextual registration*	·	Registration (including descriptions) are arbitrarily **contextual** (deictic/indexical, relative to conceptual scheme, etc.,) at arbitrary scale—not just "within sentences (or other complexes). It would thus be fully expected for a Δ calculus system to have structures analogous to such English phrases as *I*, *you*, *my*, *today*, *local*, *John*, *recently*, *transitive*, *theoretical,* etc.
P4)	*Dynamic (as well as dynamical) registration*	·	Δ calculus descriptions can not only be used to register temporal phenomena (i.e., be dynamical) but can *themselves* be temporal (i.e., dynamic). Cf. not only clocks, meters, sundials, etc., but even rhythmical patterns, oscillations, etc.
P5)	*Non-conceptual content*	·	While some expressions may register their subject matters in terms of "classical ontology" (objects exemplifying properties, standing in relations, grouped in sets, and arrayed in states of affairs), the Δ calculus is not itself committed to such registration, and supports others as well (such as Strawsonian "feature-placing")
P6)	*Metaphysical holism*	·	Rather than assume that the world is assembled from atomic or elemental parts, the underlying metaphysical assumption is that the world is whole, and that descriptions register parts of it under normatively-governed purposes.
P7)	*Meaning as (partially) use*	·	It is not assumed that descriptions register independently of how they are used, nor that their significance derives wholly from how they are used. Rather, use is (in general) viewed as a *partial determinate* of meaning.
P8)	*Registration*	·	… old version was crap; needs to be rewritten …
P9)	*Reflection*	·	The Δ calculus is reflective as well as recursive.[9] A kernel (syncategorematic) mechanism is provided with which to indicate that an expression[10] denotes, refers to, or "mentions" one or more Δ calculus structures, operations & interpretations—though what exactly is thereby mentioned (type, token, meaning, use, etc.) depends on how it is registered. With these reflective capacities, the Δ default (kernel) operations & interpretations of Δ calculus structures can be overridden at will, providing that such overriding can itself (ultimately) be described in kernel terms.
P10)	*Fusion*	·	The Δ calculus structural field (implicitly) **fuses**, as much as possible, structures that "mean" the same thing with respect to the concepts & types in terms of which they register their subject matters. ⟦This is effectively a consequence of the principle of perspectival identity; saying it properly is complex.⟧ Achieving fusion will require (substantial) computational horsepower underlying any implementation.
P11)	*Formality*	·	In spite of being a well-defined computational calculus, the Δ calculus is intended to be thoroughly "non-formal" under a variety of meanings of that term. I believe that any attempt to develop a set-theoretically based model theory for a Δ calculus system, or to prove its fundamental soundness &/or completeness, will be based on profound misunderstanding.[11]
P12)	*Interpretation*	·	It is traditional to view formal calculi as "uninterpreted" systems of marks, with issues of semantic interpretation left outside the realm of the calculus per se, although

[9]Giving the user unprecedented control over the structure, operation and interpretation semantics of all described (constructed & kernel).

[10]For now I am using 'expression' for an arbitrary Δ calculus structure; I don't know whether that will last.

[11]Is this true? I am not yet entirely sure. In some ways it seems as if it cannot be, because if the system can be implemented, once could construct a model of what it is doing. So…what is it that I mean? Possibly, that no Δ calculus identity can be absorbed into the set-theoretic model?

			in different calculi the kernel operations are typically defined with respect to (something like) a specific interpretation or interpretation schema (formal logic being the most extreme, being defined wrt an interpretation scheme of objects, properties, etc.—in some peoples' minds challenging its claim even to be a calculus). The Δ calculus, in contrast, *includes* an account of its own interpretation, in terms of which kernel operations are defined and reflective facilities described. As much as is effably possible, that is, Δ calculus is intended to embody a particular ontological/metaphysical view.
P13)	*Differentiation & Abstraction*	·	Δ calculus's approach to identity is based on a "fan-in/fan-out" conception of (something like[12]) abstraction/concretization, in which regions of the world are gathered together and taken as unities or singularities for some purposes, and divided into pluralities for others. Notions of sets vs. their members, parts vs. their wholes, abstract entities vs. their concrete exemplars, types vs. their tokens or instances, etc., are all characterized as "differentiations" of "fan-outs" on this basic model.[13]
P14)	*Physicality and efficacy*	·	Notions of locality, accessibility, etc. in Δ calculus (i.e., those relations that can lead to things happening in unit time) are based on concrete, physical connectivity & connection via effective properties. There is no notion of syntax, per se, but rather of (spatio-temporal) *effective immediacy*.[14]

III • Δ calculus Architecture

To be written. The principles are criteria to meet; they say nothing about how to do so. There should also be a section on the basic Δ calculus design, at the very highest level: the notion of the (passive but demandingly-defined structural field), etc.

IV • Other things to mention?

— "Registration" as the basic intentional notion
— …

V • Examples

Are these interesting?

1. Document annotation and reference
   a. Whole: translations × editions × types/tokens
   b. Internal: points and regions × full document grammar (cf. IA / AOS Text styles)
   c. Evolution: "this paragraph is much improved"; document reference across evolution
2. File synchronization: copy of a backup file (that gets lost);
3. Search & replace / undo (multiple varieties) / "again"
4. Translation?
5. Small examples
   a. Multiple "John"s
   b. 'Aluminum'/'aluminium' (on different web pages)

[12]Only "something like" because it is classically assumed that "abstract" individuals are not concrete, whereas in Δ CALCULUS ontology/metaphysics, all individuals are based on an act of abstraction. Because what is registered is not the "abstraction," but that which is gathered together as a unity, there is no lack of concreteness in the "abstracted" individual.
[13]Or should the term be 'fans-out'?
[14]Whether I should call this "efficacy" or "effectiveness" I am not sure.

VI • Comparison Table

…Complete out of date … here only as a place-holder that something like this would be worth generating…

Property	Calculus	Decal	Algebra	Differential Calculus	Set theory	l-calculus	Formal Logic
1	Perspectival identity	✔					
2	Deferential semantics	✔	✔	✔	✔	✔	✔
3	Contextual registration	✔					
4	Dynamic registration	✔					
5	Non-conceptual content	✔	✔				
6	Holism	✔					
7	Use	✔					

1.